## Term Information

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|---|---|
| **Effective Term** | Spring 2023 |

## General Information

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|---|---|
| **Course Bulletin Listing/Subject Area** | Mathematics |
| **Fiscal Unit/Academic Org** | Mathematics - D0671 |
| **College/Academic Group** | Arts and Sciences |
| **Level/Career** | Graduate |
| **Course Number/Catalog** | 6131 |
| **Course Title** | Introduction to Representation Theory |
| **Transcript Abbreviation** | Intro Repres Th |
| **Course Description** | Finite groups and representations; Irreducible representations; Character theory; Symmetric groups; Symmetric functions; Associative algebras; Quivers. |
| **Semester Credit Hours/Units** | Fixed: 3 |

## Offering Information

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|---|---|
| **Length Of Course** | 14 Week, 12 Week |
| **Flexibly Scheduled Course** | Never |
| **Does any section of this course have a distance education component?** | No |
| **Grading Basis** | Letter Grade |
| **Repeatable** | No |
| **Course Components** | Lecture |
| **Grade Roster Component** | Lecture |
| **Credit Available by Exam** | No |
| **Admission Condition Course** | No |
| **Off Campus** | Never |
| **Campus of Offering** | Columbus |

## Prerequisites and Exclusions

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|---|---|
| **Prerequisites/Corequisites** | Math 5101 or 5111, or grad standing, or permission of the instructor |
| **Exclusions** | |
| **Electronically Enforced** | No |

## Cross-Listings

**Cross-Listings**

## Subject/CIP Code

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|---|---|
| **Subject/CIP Code** | 27.0101 |
| **Subsidy Level** | Doctoral Course |
| **Intended Rank** | Masters, Doctoral |

## Requirement/Elective Designation

The course is an elective (for this or other units) or is a service course for other units

## Course Details

**Course goals or learning objectives/outcomes**

- Acquire a strong grasp of foundational techniques of representation theory

**Content Topic List**

- Fundamentals of representation theory.
- Representations of finite groups.
- Symmetric group and symmetric functions.
- Finite-dimensional associative algebras.
- Artin-Wedderburn theorem.
- Quiver representations.

**Sought Concurrence**

No

## Attachments

- RepresentationTheory-Syllabus.pdf: Math 6313 Sample Syllabus

*(Syllabus. Owner: Kerler,Thomas)*

## Comments

## Workflow Information

| Status | User(s) | Date/Time | Step |
|---|---|---|---|
| Submitted | Kerler,Thomas | 12/23/2021 11:02 PM | Submitted for Approval |
| Approved | Husen,William J | 12/24/2021 09:33 AM | Unit Approval |
| Approved | Vankeerbergen,Bernadette Chantal | 01/06/2022 03:17 PM | College Approval |
| Pending Approval | Cody,Emily Kathryn<br>Jenkins,Mary Ellen Bigler<br>Hanlin,Deborah Kay<br>Hilty,Michael<br>Vankeerbergen,Bernadette Chantal<br>Steele,Rachel Lea | 01/06/2022 03:17 PM | ASCCAO Approval |

**Math 6131** Syllabus **Spring 2023**

# Introduction to Representation Theory

## Instructor and Class Information

| | |
|---|---|
| **Lecturer:** | **Course Num.:** |
| **Office:** | **Lecture Room:** |
| **Phone:** | **Lecture Times:** |
| **Email:** | **Office Hours:** |

## About Course Goals

### Format

The course will meet three times a week for 55 minutes each meeting. Instruction will be mainly by lectures delivered by the instructor. It may also include occasional in-class discussion as well as short student presentations.

### Description & Goals

This course will serve as an introduction to the techniques of representation theory, its central problems, foundational results and applications. Representation theory provides systematic tools to investigate symmetries of a system (of differential equations, combinatorial structures, physical observables or geometric spaces etc). Thus familiarity with its methods is extremely useful to students aiming to be research mathematicians with interests ranging from number theory and combinatorics to geometry, probability and mathematical physics.

### Prerequisites

Math 5101 or 5111, or grad standing, or permission from the instructor.

## Textbook

### Main References

Pavel Etingof, Oleg Golnerg, Sebastian Hensel, Tiankai Liu, Alex Schwendner, Dmitry Vaintrob, Elena Yudovina (with historical interludes by Slava Gerovitch): *"Introduction to Representation Theory"*. AMS, 2011. ISBN: 0821853811.

William Fulton and Joe Harris: *"Representation Theory: a first course"*. Springer, 1991. ISBN: 0387974954.

Jean-Pierre Serre: *"Linear representations of finite groups"*. Springer, 1977. ISBN: 0387901906.

### Additional References

Amritanshu Prasad: *"Representation theory: a combinatorial viewpoint"*. Cambridge University Press, 2015. ISBN: 1107082056.

Andrei Zelevinsky: *"Representations of finite classical groups: a Hopf algebras approach"*. Lecture Notes in Mathematics 869. Springer, 1981. ISBN: 3540108246.

## Assessments

### Homework Assignments

There will be weekly homework assignments.

### Exams

There will be (at least) one midterm exam.

### Final Project

This course will have a final project instead of a final exam. The final project is a more extensive written assignment that will draw on techniques acquired throughout the semester. Final projects will be due at the beginning of the finals' week.

### Class Participation and Attendance

Although attendance is not regularly monitored frequent absences are likely to be noted and may factor into the grade in borderline cases.

## Grading

### Course Score

A course score will be computed from the above assessments. For instance, Homework assignments: 50% Mid term: 20% and Final project 30%.

### Letter Grades

Letter grades will be determined based on the course score.

## Weekly Schedule

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| **Week 1** | Associative algebras. Representations. Examples. Operations on representations: direct sums, tensor products, homomorphisms, duals. Sub and quotient representations. |
| **Week 2** | Semisimplicity. Irreducible vs indecomposable representations. Schur's lemma. |
| **Week 3** | Finite groups. Maschke's complete reducibility theorem. Group algebra. |
| **Week 4** | Group determinant. Frobenius' character theory. Orthogonality and character table. |
| **Week 5** | Applications: counting lemmas, algebraic numbers and characters, Burnside's theorem. |
| **Week 6** | Induction and restriction. Frobenius reciprocity. Mackey's double coset theorem. |
| **Week 7** | Symmetric group. Partitions. Conjugacy classes. Basic counting results. |
| **Week 8** | Ring of symmetric functions. Elementary, complete and power sum symmetric functions. |
| **Week 9** | Frobenius character map. Schur polynomials. Young lattice and tableau. |
| **Week 10** | Jacobi-Trudi identity. Robinson-Schensted-Knuth correspondence. |
| **Week 11** | Generalities on associative algebras. Radical. Semisimple algebras. Artin-Wedderburn theorem. Density theorem. Krull-Schmidt theorem. |
| **Week 12** | Path algebra of a quiver. Quiver representations. Simple, projective and injective reps. |
| **Week 13** | Quivers of finite type. Gabiel's theorem. Bernstein-Gelfand-Ponomarev functors. |
| **Week 14** | Representations of Jordan quiver. |

## General Policies

### Academic Misconduct

Academic integrity is essential to maintaining an environment that fosters excellence in teaching, research, and other educational and scholarly activities. Thus, The Ohio State University and the Committee on Academic Misconduct (COAM) expect that all students have read and understand the University's Code of Student Conduct, and that all students will complete all academic and scholarly assignments with fairness and honesty. Students must recognize that failure to follow the rules and guidelines established in the University's Code of Student Conduct and this syllabus may constitute Academic Misconduct.

The Ohio State University's Code of Student Conduct (Section 3335-23-04) defines academic misconduct as: Any activity that tends to compromise the academic integrity of the University, or subvert the educational process. Examples of academic misconduct include (but are not limited to) plagiarism, collusion (unauthorized collaboration), copying the work of another student, and possession of unauthorized materials during an examination. Ignorance of the University's Code of Student Conduct is never considered an excuse for academic misconduct, so I recommend that you review the Code of Student Conduct and, specifically, the sections dealing with academic misconduct.

If I suspect that a student has committed academic misconduct in this course, I am obligated by University Rules to report my suspicions to the Committee on Academic Misconduct. If COAM determines that you have violated the University's Code of Student Conduct (i.e., committed academic misconduct), the sanctions for the misconduct could include a failing grade in this course and suspension or dismissal from the University.

### Disability Services

The university strives to make all learning experiences as accessible as possible. In light of the current pandemic, students seeking to request COVID-related accommodations may do so through the university's request process (slds.osu.edu/covid-19-info/covid-related-accommodation-requests/), managed by Student Life Disability Services. If you anticipate or experience academic barriers based on your disability (including mental health, chronic, or temporary medical conditions), please let me know immediately so that we can privately discuss options. To establish reasonable accommodations, I may request that you register with Student Life Disability Services. After registration, make arrangements with me as soon as possible to discuss your accommodations so that they may be implemented in a timely fashion. SLDS contact information: slds@osu.edu; 614-292-3307; slds.osu.edu; 098 Baker Hall, 113 W. 12th Avenue.